Efficient Surrogate Optimization for Payment Cost Co-Optimization with Transmission Capacity Constraints

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***Abstract*—Most ISOs in the US minimize the total bid cost and then settle the market based on locational marginal prices. Minimizing the total bid cost, however, may not lead to maximizing the social welfare. Studies indicate that for energy only, payment cost minimization (PCM) leads to reduced payments for a given set of bids, and the "hockey-stick" bidding a less likely to occur. Since co-optimization of energy with ancillary services leads to a more efficient allocation, it is important to solve PCM co-optimization while considering transmission capacity constraints for a comparison with other auction mechanisms. In this paper, PCM is formulated using price definition system-wide constraints. This problem formulation introduces difficulties such as nonlinearity, non-separability and complexity of convex hull. To overcome these difficulties, the nonlinear terms are linearized thereby allowing to be solved by using branch-and-cut. At the same time, the linearization is performed in a way that the "surrogate optimality condition" is satisfied thereby allowing the problem to be solved efficiently.**

***Index Terms*—Payment Cost Minimization, Transmission Capacity Constraints, Ancillary Services, Lagrangian Relaxation and Surrogate Optimization, Branch-and-cut**

I. INTRODUCTION

Most ISOs in the US use bid cost minimization (BCM) to minimize the total bid cost and then settle the market based on uniform market clearing prices (MCPs) or congestion-adjusted locational marginal prices (LMPs), which are byproducts of the minimization. The minimized total bid cost, however, could be significantly different from the total payment cost. This is a serious issue because minimization of the total bid cost may not lead to maximization of the social welfare [1] since bids are generally different from production costs. An alternative auction mechanism that minimizes the total payment cost directly, payment cost minimization (PCM), has been considered in the literature [2].

Unlike those in BCM, prices are decision variables in PCM and need to be appropriately defined. In PCM with transmission capacity constraints, prices have been defined using system-wide price definition constraints [3] and [4] for each bus. Such constraints, however, can lead to defining multiple marginal generators that are eligible to set prices.

Other difficulties that PCM introduces are associated with methodological challenges for Lagrangian relaxation [2] and branch-and-cut [4] and [5]. While Lagrangian relaxation can exploit separability and branch-and-cut can exploit linearity of BCM, price definition constraints in PCM cause complications such as nonlinearity, non-separability and complexity of the convex hull. Preliminary results to overcome these difficulties have been presented in [3], [4], [5] and [6].

Building on the results of [3], the nonlinear PCM co-optimization with transmission capacity constraints is presented in Section III. In the new formulation, LMP are characterized by marginal generators and price setting eligibility criteria are defined using shift factors, that is, by the possible congestion impact that a particular generator can introduce into the transmission network. To overcome methodological challenges mentioned before, surrogate optimization framework and the novel linearization procedure are presented in Section IV. In the method, the nonlinear relaxed problem is linearized thereby allowing to be solved by using branch-and-cut. At the same time, the linearization is performed in such a way that the "surrogate optimality condition" is satisfied thereby allowing the problem to be solved efficiently. In addition, terms in the relaxed problem involving shift factors may lead to excessive computational effort and will be regrouped. Numerical examples presented in Section V demonstrate that the new surrogate optimization method can frequently obtain surrogate multiplier-updating directions that form an acute angles toward the optimal multipliers. Large numerical examples demonstrate that the new method converges within a reasonable CPU time and obtains feasible solutions with reasonable quality.

II. LITERATURE REVIEW

Many studies have been conducted on solving difficult mixed-integer optimization problems. Such problems can often be regarded as simple ones subject to complicating constraints [7]. The relaxation of such constraints by Lagrange multipliers has been a successful approach toward obtaining near-optimal solutions efficiently [7] and [8]. After the relaxed problem is optimized, Lagrange multipliers are adjusted based on the levels

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of constraint violation traditionally by using subgradient directions and appropriately chosen stepsizes [9]. The subgradient method has been an extensive research topic and multiple convergent variations have been reported over the decades, such as the subgradient level method [10] and [11].

While many optimization methods (Lagrangian relaxation and branch-and-cut) have been efficient for solving BCM, there has been limited research on solution methodologies for PCM. While Augmented Lagrangian and surrogate optimization [2] has been successful for solving PCM, separability cannot be exploited because price definition constrains couple individual generator subproblems. Since PCM without transmission capacity constraints can be formulated in a linear form, [4] solves PCM by using branch-and-cut together with the objective switching method. Nevertheless, since branch-and-cut handles constraints globally, unlike Lagrangian relaxation, local characteristics cannot be exploited, cutting planes that are valid across price definition constrains are difficult to obtain [4] and the method can suffer from slow convergence. To efficiently exploit local characteristics and linearity of PCM, a combination of Lagrangian relaxation, surrogate optimization and branch-and-cut has been developed [5], [6] and [12]. While PCM cannot be decomposed into individual generator subproblems within the Lagrangian relaxation framework, multiplier-updating directions can be efficiently obtained within surrogate optimization by solving one generator subproblem at a time by using branch-and-cut until the "surrogate optimality condition" [8] is satisfied.

Convergence of the surrogate optimization method has been proved in [8], and the method has shown to provide a measure of solution quality by generating a valid lower bound to the feasible cost. Nevertheless, convergence proof assumes the knowledge of the optimal dual value, which is unknown. In standard subgradient methods (e.g., subgradient level method), optimal dual value can be adaptively estimated to overcome the convergence difficulties [7] and [9]. For the surrogate subgradient method, however, the surrogate dual value may lose the lower bound property thereby failing to set the appropriate "target level" and failing to lead to convergence.

To overcome this difficulty, stepsizes has been selected in a way that distances between Lagrange multipliers at consecutive iterations decrease, leading Lagrange multipliers converge to a unique limit. At the same time, stepsizes are kept sufficiently large so that the algorithm does not terminate prematurely [12]. While PCM without transmission capacity constraints have been solved efficiently by using a combination Lagrangian relaxation, surrogate optimization and branch-and-cut [5], and [6], when transmission capacity constraints are included, the PCM formulation is nonlinear and cannot be directly solved by using the combined method. Preliminary results to resolve the nonlinearity issues presented in [3] are based on iterative linearization by fixing and updating prices.

III. Problem Formulation

This section presents PCM co-optimization with transmission capacity constraints while developing the price definition by using marginal concept and KKT optimality conditions. The problem formulation is based on [3] and is extended to characterize generators eligible to set prices.

PCM Problem Description

The objective of PCM co-optimization is to minimize the total customer payment cost, consisting of the total cost on energy, spinning reserve and start-up cost. In a network with I nodes connected by L transmission lines, M_i supply generators at each node $i \in I$ submit energy and spinning reserve bids. Each bid $m \in M_i$ indexed by (i,m) consists of energy and spinning reserve bidding prices $c_{(i,m)}^{E(S)}$, startup costs $S_{(i,m)}$, maximum and minimum generation levels $p_{(i,m)max}$, $p_{(i,m)min}$ and maximum levels for spinning reserve $p_{(i,m)max}^{S}$. Minimum up- and down-times and ramp-rate constraints are not considered in this paper.

Generation Capacity Constraints

The energy and spinning reserve decision variables corresponding to each generator (i,m) are denoted by $p_{(i,m)}^{E}(t)$ and $p_{(i,m)}^{S}(t)$, respectively. Energy and spinning reserve allocation status binary decision variables are denoted by $x_{(i,m)}^{E}(t)$ and $x_{(i,m)}^{S}(t)$. The relationship between these decision variables can be summarized as the following individual generator capacity constraints:

$$x_{(i,m)}^{E}(t)\,p_{(i,m)\min} \le p_{(i,m)}^{E}(t) \le x_{(i,m)}^{E}(t)\,p_{(i,m)\max}\,, \quad (1)$$

$$0 \le p_{(i,m)}^{S}(t) \le x_{(i,m)}^{S}(t)\,p_{(i,m)\max}^{S}\,, \quad (2)$$

$$p_{(i,m)}^{E}(t) + p_{(i,m)}^{S}(t) \le p_{(i,m)\max}\,. \quad (3)$$

The startup cost $S_{(i,m)}$ is incurred if and only if the generator has been turned "on" from an "off" state at hour t

$$u_{(i,m)}(t) \ge x_{(i,m)}(t) - x_{(i,m)}(t-1)\,. \quad (4)$$

Power Flow Constraints

The power flow $f_{(b1,b2)}(t)$ in a line connecting buses b_1 and b_2 at hour t can be expressed as a linear combination of net nodal injection of energy at hour t [13]:

$$f_{(b_1,b_2)}(t) = \sum_{i=1}^{I} a_{(b_1,b_2)}^{i} \cdot \left(\sum_{m=1}^{M_i} p_{(i,m)}^{E}(t) - P_i^{DE}(t) \right). \quad (5)$$

Shift factors a_l^i denote power transmitted through line l when 1MW is injected at bus i and withdrawn at a reference bus. Shift factors depend on the network and transmission line structure. Power flows in any line cannot exceed the transmission capacity $f_{l\max}$ which for simplicity is set to be the same for each direction

$$-f_{(b_1,b_2)\max} \le f_{(b_1,b_2)}(t) \le f_{(b_1,b_2)\max}\,. \quad (6)$$

Demand and Spinning Reserve Requirement Constraints

A set of generators satisfying (1)-(3) is selected to satisfy energy nodal load levels $P_i^{DE}(t)$ either locally or by transmitting power through transmission lines. The total power generated should be equal to the system demand at each hour t:

$$\sum_{i=1}^{I}\sum_{m=1}^{M_i} p_{(i,m)}^{E}(t) = \sum_{i=1}^{I} P_i^{DE}(t)\,. \quad (7)$$

A set of generators satisfying (1)-(3) is also selected to satisfy the total spinning reserve requirements $P_i^{DS}(t)$:

$$\sum_{i=1}^{I}\sum_{m=1}^{M_i} p_{(i,m)}^{S}(t) \ge P^{DS}(t)\,. \quad (8)$$

Locational Marginal Prices and Market Clearing Prices

Locational marginal energy prices and the market clearing spinning reserve price for node i are denoted by $LMP_i^E(t)$ and $MCP^S(t)$ respectively. As discussed in [3], LMPs are marginal prices that reflect the cost of production of an additional εMW of power. In general, generators with power levels neither at p_{min} nor at p_{max} can produce an additional εMW of power. In the marginal case, an additional εMW increase of the system demand can lead an expensive generator to be brought online at p_{min}, and to reduction of power levels of other generators. In such cases, the generator at p_{min} sets a price. The price setting eligibility criteria have been captured in [3] by binary decision variables $y_{(i,m)}{}^E$ that depend on $x_{(i,m)}{}^E$ and $p_{(i,m)}{}^E$. However, when using such price definition for each node, there can potentially be several marginal generators in the transmission network.

To overcome this issue, marginal units are captured by using shift factors. If the demand $P_j^{DE}(t)$ increases by εMW, a local generator can generate extra εMW to meet the demand $P_j^{DE}(t)+\varepsilon$. If a generator is locational, its power generated can potentially increase power flows in congested lines. In this case, other generators have to curtail their generation levels by a certain amount δMW to keep the power balance in the system. If a generator (i,m), can then generate an additional $(\varepsilon+\delta)$MW in such a way that a δ amount of power can be curtained by other units without violating transmission capacity constraints, generator (i,m) can set the price for bus i. If a $p_{(i,m)\min}$ unit can meet requirements from above (power can be redistributed among units) it is eligible for price setting ($y_{(i,m)}{}^E = 1$) if and only if power flow increase in a congested line caused by generator (i,m) is the lowest. That is, $a_l^{(i,m)}>0$ is lower than other shift factors corresponding to other online generators. LMPs are defined as:

$$LMP_i^E \geq c_{(i,m)}^E \cdot y_{(i,m)}^E. \tag{9}$$

To obtain the relationship between the LMP at a bus i and the LMP at the reference bus, multipliers $\{\gamma_{\max}, \gamma_{\min}\}$ relaxing transmission capacity constraints (6), and multipliers λ^E relaxing the system demand constraint (7) are introduced, and the KKT conditions are derived based on the following Lagrangian:

$$\begin{aligned} L &= \sum_{i=1}^{I} LMP_i^E \cdot \sum_{m=1}^{M_i} p_{(i,m)}^E + MCP^S \cdot \sum_{i=1}^{I}\sum_{m=1}^{M_i} p_{(i,m)}^S \\ &+ \sum_{l=1}^{L} \gamma_{l\max} \cdot \left[\sum_{i=1}^{I} a_l^i \cdot \left(\sum_{m=1}^{M_i} p_{(i,m)}^E - P_i^D \right) - f_{l\max} \right] \\ &+ \sum_{l=1}^{L} \gamma_{l\min} \cdot \left[-\sum_{i=1}^{I} a_l^i \cdot \left(\sum_{m=1}^{M_i} p_{(i,m)}^E - P_i^D \right) - f_{l\max} \right] \\ &+ \lambda^E \left(\sum_{i=1}^{I} P_i^{DE} - \sum_{i=1}^{I}\sum_{m=1}^{M_i} p_{(i,m)}^E \right). \end{aligned} \tag{10}$$

The KKT conditions can be derived by taking a partial derivative of (10) with respect to $p^E{}_{(i,m)}$:

$$\frac{\partial L}{\partial p_{(i,m)}^E} = LMP_i^E - \lambda^E + \sum_{l=1}^{L} (\gamma_{l\max} - \gamma_{l\min}) \cdot a_l^i = 0. \tag{11}$$

For the reference bus, $a_l^i=0$, thus we have

$$LMP_{ref}^E = \lambda^E. \tag{12}$$

The relationship between an arbitrary $LMP^E{}_i$ and $LMP^E{}_{ref}$ is:

$$LMP_i^E = LMP_{ref}^E + \sum_{l=1}^{L} a_l^i (\gamma_{l\min} - \gamma_{l\max}). \tag{13}$$

For the spinning reserve, each generator can provide spinning reserve starting from 0MW rather than from p_{min}. Thus, the highest spinning reserve price is used:

$$MCP^S \geq c_{(i,m)}^S \cdot x_{(i,m)}^S. \tag{14}$$

The objective of PCM co-optimization is to minimize the total customer payment cost:

$$\sum_{i=1}^{I}\sum_{m=1}^{M_i} \left(LMP_i^E \cdot p_{(i,m)}^E + MCP^S \cdot p_{(i,m)}^S + S_{(i,m)} u_{(i,m)} \right). \tag{15}$$

IV. Solution Methodology

This section presents the solution methodology for solving PCM co-optimization with transmission capacity constraints. The method is based on the combination of the Lagrangian relaxation, surrogate optimization and branch-and-cut. While the PCM problem is non-separable and nonlinear, in the combined method, the nonlinear relaxed problem substituted by a linear function thereby allowing to be solved by using branch-and-cut. At the same time, the linear function is chosen in such a way the "surrogate optimality condition" is satisfied thereby allowing the problem to be solved efficiently.

Surrogate Optimization

In Lagrangian relaxation, selected constraints are relaxed. In PCM, transmission capacity constraints (6) are selected for the following reasons: 1) By relaxing transmission capacity constraints, congestion prices can be captured; 2) Computational effort required to handle transmission capacity constraints can be often prohibitive. When constraints (6) are relaxed, Lagrangian becomes:

$$\begin{aligned} L &= \sum_{i=1}^{I} \left(LMP_i^E \cdot \sum_{m=1}^{M_i} p_{(i,m)}^E + MCP^S \cdot \sum_{m=1}^{M_i} p_{(i,m)}^S \right) + \\ &\sum_{l=1}^{L} \gamma_{l\max} \cdot (f_l - f_{l\max}) + \sum_{l=1}^{L} \gamma_{l\min} \cdot (-f_l - f_{l\max}). \end{aligned} \tag{16}$$

Since Lagrangian (16) is difficult to optimize, surrogate optimization [8] will be used, in which surrogate directions are calculated after obtaining an approximate optimum of (16) satisfying the following surrogate optimality condition [8]:

$$\tilde{L}(\gamma^k, x^k) < \tilde{L}(\gamma^k, x^{k-1}), k = 1,2,\ldots. \tag{17}$$

The multipliers are updated as

$$\gamma^{k+1} = \gamma^k + c^k \tilde{g}(x^k), \; k = 0,1,\ldots. \tag{18}$$

Here x^k collectively denotes decision variables. In terms of PCM, surrogate directions are obtained as

$$\tilde{g}_{(\min)\max}\left(p_{(i,m)}^{E,k}\right) = \mp f_l - f_{l\max}. \tag{19}$$

In order to update stepsizes in (18), surrogate optimization with fixed-point mapping method developed in [10] will be used. The key idea is to obtain stepsizes such that distances between multipliers at consecutive iterations decrease, i.e.,

$$\left\| \gamma^{k+1} - \gamma^k \right\| = \alpha_k \left\| \gamma^k - \gamma^{k-1} \right\|, \; 0 < \alpha_k < 1, \; k = 1,2,\ldots \tag{20}$$

Stepsizes satisfying (20) can be derived using (19). Indeed (19) and (20) imply

$$\left\| c^k \tilde{g}\left(p_{(i,m)}^{E,k}\right) \right\| = \alpha_k \left\| c^{k-1} \tilde{g}\left(p_{(i,m)}^{E,k-1}\right) \right\|, \; 0 < \alpha_k < 1, \; k = 1,2,\ldots \tag{21}$$

Since c^k and c^{k-1} are positive scalars, (21) implies

$$c^k = \alpha_k \frac{c^{k-1} \left\| \tilde{g}\left(p_{(i,m)}^{E,k-1}\right) \right\|}{\left\| \tilde{g}\left(p_{(i,m)}^{E,k}\right) \right\|}, \; 0 < \alpha_k < 1, \; k = 1,2,\ldots. \tag{22}$$

Here the surrogate norm is defined as

$$\left\| \tilde{g}\left(p_{(i,m)}^{E,k}\right) \right\|^2 = \left\| \tilde{g}_{\min}\left(p_{(i,m)}^{E,k}\right) \right\|^2 + \left\| \tilde{g}_{\max}\left(p_{(i,m)}^{E,k}\right) \right\|^2. \tag{23}$$

The key idea is to make sure that Lagrange multipliers converge to a unique limit and keep stepsizes sufficiently large so that the algorithm will not terminate prematurely.

Linearization

Surrogate optimization has been efficient to solve linear mixed-integer problems [5], [6] and [12] when combined together with branch-and-cut. To exploit linearity, nonlinear terms will be linearized by selecting one variable as the decision variable while fixing other variables such that the resulting terms are linear. The resulting linear problem will then be optimized and fixed values will be updated. For example, a product of two decision variables $x\cdot y$ can be linearized as $\frac{1}{2}x_k\cdot y+\frac{1}{2}x\cdot y_k$. After obtaining (x^{k+1},y^{k+1}), (x^k,y^k) are updated to obtain $x^{k+1}\cdot y^{k+1}$. Thus, the Lagrangian (16) considering the KKT conditions (13) can be linearized as

$$\begin{aligned}\hat{L}\left(\gamma, LMP_i^{E,k}, LMP_i^{E}, p_{(i,m)}^{E,k}, p_{(i,m)}^{E}\right) &= \frac{1}{2}\sum_{i=1}^{I} LMP_i^{E,k}\cdot\sum_{m=1}^{M_i} p_{(i,m)}^{E} + \\ &\frac{1}{2}\sum_{i=1}^{I}\left(LMP_{ref}^{E} + \sum_{l=1}^{L} a_l^i(\gamma_{l\min}-\gamma_{l\max})\right)\cdot\sum_{m=1}^{M_i} p_{(i,m)}^{E,k} + \\ &\frac{1}{2}MCP^{S,k}\cdot\sum_{i=1}^{I}\sum_{m=1}^{M_i} p_{(i,m)}^{S} + \frac{1}{2}MCP^{S}\cdot\sum_{i=1}^{I}\sum_{m=1}^{M_i} p_{(i,m)}^{S,k} + \\ &\sum_{l=1}^{L}\gamma_{l\max}\cdot\left(f_l - f_{l\max}\right) + \sum_{l=1}^{L}\gamma_{l\min}\cdot\left(-f_l - f_{l\max}\right).\end{aligned} \qquad (24)$$

After (24) is approximately minimized, and $LMP_i^{E,k+1}$ with $p_{(i,m)}^{E,k+1}$ are obtained, the values $LMP_i^{E,k}$ and $p_{(i,m)}^{E,k}$ are updated:

$$\begin{aligned}&\tilde{L}\left(\gamma^{k+1}, LMP_i^{E,k}, LMP_i^{E,k+1}, p_{(i,m)}^{E,k}, p_{(i,m)}^{E,k+1}\right)\Big|_{\substack{p_{(i,m)}^{E,k}=p_{(i,m)}^{E,k+1}\\ LMP_i^{E,k}=LMP_i^{E,k+1}}} \\ &= \tilde{L}\left(\gamma^{k+1}, LMP_i^{E,k+1}, p_{(i,m)}^{E,k+1}\right).\end{aligned} \qquad (25)$$

Since the surrogate optimization method with fixed-point mapping converges and provides a valid lower bound [12], and the surrogate optimality condition of the new method is satisfied[1], the new method also converges and generates a valid lower bound to the feasible cost. At convergence, simple heuristics can be used to obtain the feasible solution. Given that complementary slackness conditions are satisfied at optimum

$$\gamma_{l\max}\cdot\left[f_l - f_{l\max}\right]=0,\ \gamma_{l\min}\cdot\left[-f_l - f_{l\max}\right]=0 \qquad (26)$$

only transmission capacity constraints corresponding to $\gamma_{\min(\max)}>0$ are to be considered when searching for a feasible solution.

Problem Formulation Simplification

When solving large instances, computations of power flows (5) can be expensive. To overcome this difficulty, (5) can be substituted in (24) thereby reducing the number of constraints and computational requirements. Summations of shift factors to be performed for the resulting relaxed problem can still be costly. Therefore, terms in (24) are reformulated as one summation:

$$\begin{aligned}\hat{L}\left(\gamma, LMP_i^{E,k}, p_{(i,m)}^{E,k}\right) &= \frac{1}{2}\sum_{i=1}^{I}\sum_{m=1}^{M_i}\Big(LMP_i^{E,k}\cdot p_{(i,m)}^{E} + LMP_{ref}^{E}\cdot p_{(i,m)}^{E,k} + \\ &\frac{1}{2}MCP^{S,k}\cdot p_{(i,m)}^{S} + \frac{1}{2}MCP^{S}\cdot p_{(i,m)}^{S,k}\Big) - \sum_{l=1}^{L}\left(\gamma_{l\max}+\gamma_{l\min}\right)\cdot f_{l\max} + \\ &\sum_{l=1}^{L}\left(\sum_{i=1}^{I} a_l^i\left(\gamma_{l\min}-\gamma_{l\max}\right)\cdot\left(\frac{1}{2}\sum_{m=1}^{M_i} p_{(i,m)}^{E,k} - \sum_{m=1}^{M_i} p_{(i,m)}^{E} + P_i^{D}\right)\right).\end{aligned} \qquad (27)$$

[1] If $f(x,y)=x\cdot y$ (s.t. linear $g(x,y)\le 0$) is linearized as $\frac{1}{2}x_k\cdot y+\frac{1}{2}x\cdot y_k$, then $xy < 1/2x^ky+1/2\,xy^k$ when $x\in(0,x^k)$ and $y\in(0,y^k)$. Therefore, after the linearized Lagrangian $1/2x^ky+1/2\,xy^k+\lambda g(x,y)$ is optimized, such that $1/2x^ky^{k+1}+1/2\,x^{k+1}y^k+\lambda g(x^{k+1},y^{k+1}) < x^ky^k+\lambda g(x^k,y^k)$, then $x^{k+1}y^{k+1}+\lambda g(x^{k+1},y^{k+1}) < x^ky^k+\lambda g(x^k,y^k)$.

V. NUMERICAL TESTING

The new method has been implemented by using CPLEX 12.2 on Intel® Xeon® CPU E5620 (12M Cache, 5.86 GT/s Intel® QPI) @ 2.40GHz (2 processors) and 36.00 GB of RAM. In this section two examples are considered. The small PCM problem with 6 nodes and 7 transmission lines is considered first to demonstrate that surrogate directions frequently form small acute angles with directions toward the optimal multipliers. Large-scale PCM problems with 118 buses and 186 transmission lines are then considered to demonstrate that the method is capable of efficiently solving large instances, and the method is compared with the subgradient method.

Example 1: Small Payment Cost Minimization problem.

In this example, 6 buses, 16 generators and 7 transmission lines is considered. For simplicity, only energy product is considered and start-up costs are not considered. The network topology is shown in Figure 1, and shift factors and bid data are shown in Tables 1 and 2, respectively. The maximum transmission capacity of each transmission line is 50MW.

Table 1: Shift factors for Example 1

Bus #	*Line* (1,3)	*Line* (1,2)	*Line* (2,4)	*Line* (3,4)	*Line* (3,5)	*Line* (5,6)	*Line* (4,6)
1	0.75	0.25	0.25	0.25	0	0	0
2	0.5	-0.5	0.5	0.5	0	0	0
3	0	0	0	0	0	0	0
4	0.2	-0.2	-0.2	0.6	0.2	0.2	-0.2
5	0	0	0	0.25	0.75	-0.25	0.25
6	0	0	0	0.5	0.5	0.5	0.5

Table 2: Generator data for Example 1

Generator	*Price* ($/MW)	p_{max} (MW)	p_{min} (MW)	Generator	*Price* ($/MW)	p_{max} (MW)	p_{min} (MW)
(1,1)	5	60	10	(4,1)	30	50	20
(1,2)	10	60	10	(4,2)	35	60	20
(1,3)	15	60	10	(4,3)	40	50	20
(1,4)	20	60	10	(4,4)	45	60	20
(2,1)	25	60	10	(6,1)	45	60	20
(2,2)	30	50	10	(6,2)	50	50	20
(2,3)	35	60	10	(6,3)	55	60	30
(2,4)	40	50	10	(6,4)	60	50	30

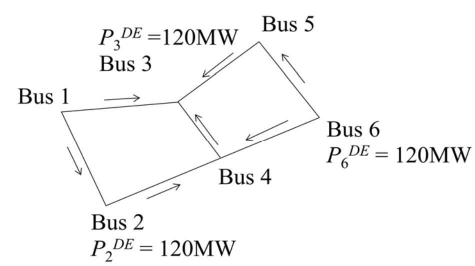

Figure 1. Transmission network for Example 1

Following the solution methodology developed in Section IV, surrogate directions and stepsizes are updated using (19) and (22) with α_k equal 0.99. The number of iterations required for convergence is 230, and the CPU time is 15.28 sec. In this example, the method obtains the optimal dual value, which equals to the optimal primal value = $12,480. Figure 2 shows the optimal power flows and optimal LMPs, and Table 3 shows optimal power generated at each generator. The trajectories of the multipliers are shown in Figure 3. Surrogate multiplier-updating directions (shown by arrows) obtained frequently form small acute angles with directions toward the optimal multipliers.

Table 3: Optimal dispatch for Example 1							
Generator	***Power* (MW)**	**Generator**	***Power* (MW)**	**Generator**	***Power* (MW)**	**Generator**	***Power* (MW)**
(1,1)	60	(2,1)	24	(4,1)	50	(6,1)	60
(1,2)	36	(2,2)	0	(4,2)	60	(6,2)	50
(1,3)	0	(2,3)	0	(4,3)	20	(6,3)	0
(1,4)	0	(2,4)	0	(4,4)	0	(6,4)	0

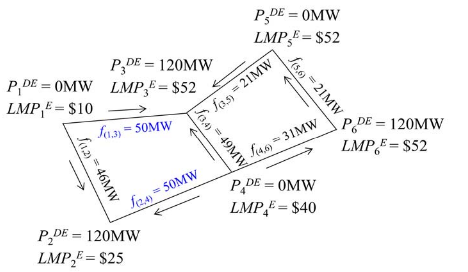

Figure 2. Optimal LMPs and power flows for Example 1

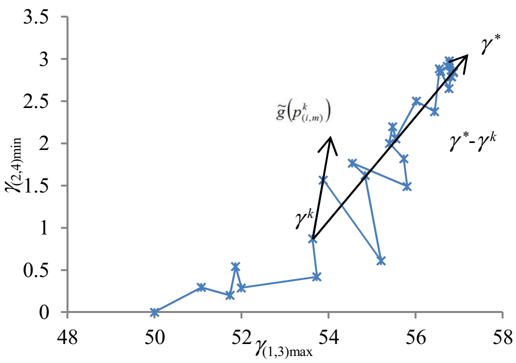

Figure 3. Trajectory of the multipliers

Example 2: Large-scale PCM co-optimization problem.

To demonstrate the efficiency of the method developed in Section IV by comparing it with the subgradient method, a 24-hour PCM co-optimization problem with 118 buses and 186 transmission lines is considered. The data are based on the data for the IEEE 118-bus system.

The problem is solved by using surrogate optimization and branch-and-cut with stepsizes (27). Numerical results are summarized in Table 4.

Instance	Table 4: Numerical results for Example 2				
	# of congested lines	***Lower Bound ($)***	***Feasible Cost ($)***	***CPU time (min)***	***Duality Gap (%)***
1	5	54,910,659	56,124,833	10	2.16334
2	10	54,820,399	56,527,603	10	3.02012
3	15	54,731,205	56,655,662	10	3.39676

Instance	Table 5: Numerical results for Example 2				
	# of congested lines	***Lower Bound ($)***	***Feasible Cost ($)***	***CPU time (min)***	***Duality Gap (%)***
1	5	54863890	56162316	~80	2.311918
2	10	54785089	56537226	~80	3.099085
3	15	54717960	56737142	~80	3.558837

In all instances, the algorithm converges within 10 minutes of CPU time. Given the congestion prices, feasible solutions are obtained using heuristics embedded in CPLEX. When using the subgradient method, CPU time required per iteration increases significantly and the total CPU time for each instance is greater than 60 minutes as shown in Table 5.

Without simplifications presented in (26), CPU time increases significantly since all power flow constraints (5) need to be computed individually.

VI. Conclusions

This paper considers PCM co-optimization problem formulation with transmission capacity constraints and develops the efficient solution methodology. PCM is formulated using price definition system-wide constraints by using shift factors to characterize marginal units. To overcome difficulties that the problem formulation introduces, such as nonlinearity, non-separability and complexity of convex hull, the problem is solved by a combination of surrogate optimization and branch-and-cut. It is demonstrated that the resulting surrogate multiplier-updating directions frequently form acute angles with directions toward optimal multipliers. Based on recently developed surrogate optimization and fixed-point mapping, the method developed in this paper converges to the optimum without requiring the optimal dual value. Large examples demonstrate that the method is better than the frequently used subgradient method and can obtain feasible solution efficiently and with reasonable quality.

VII. References

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